

# Improving the infra-red of holographic descriptions of QCD

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**ABSTRACT:** A surprisingly good holographic description of QCD can be obtained from naive five dimensional gauge theory on a truncated AdS space. We seek to improve the infra-red description of QCD in such models by using a more sophisticated metric and an action derived from string theory duals of chiral symmetry breaking. Our metric is smooth into the infra-red and the chiral condensate is a prediction of the dynamics. The theory reproduces QCD meson data at the 10% level.

**KEYWORDS:** AdS-CFT Correspondence, Duality in Gauge Field Theories, QCD.

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## 1. Introduction

The deep connections between QCD and string theory have been revived in recent years by the AdS/CFT Correspondence [1–3]. The Correspondence provides an explicit description of a strongly coupled gauge theory in terms of a weakly coupled, holographic string description. The original conjecture was for a highly supersymmetric conformal theory with only adjoint fields. Technology has since been introduced that allows supersymmetry to be broken and a running gauge coupling to be present [4–6], and the introduction of quark fields [7–9]. Confinement [10, 11] and chiral symmetry breaking [12]–[19] have been investigated using the string description.

Recently, the first attempts have been made to construct phenomenological holographic models of QCD [20, 21] (see also [22]–[31]). Surprisingly simple models consisting of gauge theory in an anti-de-Sitter space interval have turned out to provide a remarkably good description of the meson sector of QCD. These models are in many ways naive though. Amongst the criticisms that might be aimed at these models are:

- The use of an AdS geometry implicitly means that the background gauge configuration is conformal (and essentially that of large N  $\mathcal{N} = 4$  super Yang Mills).
- The existence of a mass gap is imposed by hand through the inclusion of a boundary to the space and is not the product of a running coupling.
- The fields that holographically describe the quark bilinears are included phenomenologically and there is no rigorous (string theory) realization of the construction.
- The solution for the field which describes the quark mass and condensate is also included by hand and the quark condensate is not dynamically determined in terms of either the gauge configuration or the quark mass.

- The ultra-violet of the theory does not become asymptotically free.
- The excited meson mass spectrum typically scales like the excitation number  $n$  as opposed to the  $\sqrt{n}$  scaling predicted by a simple flux tube model [32].

In spite of these objections, the models do provide a good description of the light meson sector of QCD. The clear next step is to try to alleviate some or all of these objections. In this paper we will address this task (progress has already been made in [24, 31]).

Our main tool will be to use the more rigorous AdS/CFT description of chiral symmetry breaking in [12]. Previously it has been used as a testing ground for the generic features of chiral symmetry breaking [13], but here we will massage it to a phenomenological five dimensional holographic description of QCD.

The geometry we will use is that on the surface of a D7 brane in a non-supersymmetric dilaton flow deformation of the AdS/CFT Correspondence. We review its origin in more detail in the appendix, but let us stress its benefits now

- The background gauge configuration in which the quarks live is non-supersymmetric (although not purely that of QCD) and has a running coupling.
- The mass gap is a result of the non-supersymmetric gauge configuration and the geometry relevant for quark physics is smooth at all radii or energy scales.
- The holographic dual of the quark bilinear is explicit in the string construction.
- The quark condensate is a prediction of the gauge configuration and is determined as a function of the quark mass.

These points go a considerable way towards addressing the inconsistencies of the first models. We will, however, continue to adopt the phenomenological approach with regards treating the background as describing an  $N=3$  rather than  $N \rightarrow \infty$  theory. In addition, the string theory construction can only realize a  $U(1)$  axial symmetry, and does not provide a holographic dual of the axial vector mesons. We include by hand appropriate fields to provide a non-abelian chiral symmetry and the axial vector states in the phenomenological spirit of [20, 21].

One knows that the transverse parts of the vector vector and axial axial correlators in QCD interact differently with the chiral condensate in QCD. In the gravity dual one would expect the axial and vector gauge fields to in fact see distinct metrics. We can not incorporate this effect because the string model does not provide enough information. Nevertheless the model links the quark condensate to the dynamics and smooths the infrared which should improve the description, at least in the vector sector, whilst doing no more harm in the axial sector than is done in [20, 21].

In this paper we compute with our phenomenological model the masses and decay constants for the pion and the rho and  $a_1$  vector mesons, and also the  $g_{\rho\pi\pi}$  coupling. We find that the model gives comparable predictions to the pure AdS models within 12% of the QCD values. We believe these results provide support for the robustness of the predictions of these holographic models.

The geometry we propose returns to pure AdS space in the ultra-violet, so we do not address here the absence of asymptotic freedom in the gravity description. As we pointed out recently in [33], the gravity theory should only be used up to a UV cut off, corresponding to the scale at which QCD switches from perturbative to non-perturbative behaviour. Above that cut off the gravitational dynamics must become non-perturbative with its loop corrections completely dominating the classical results. The correct UV dynamics should be encoded at that cut off by correcting the values of higher dimension operator couplings. In principle, these can be tuned in the AdS/CFT approach to produce the holographic equivalent of a perfect lattice action.

As a small example of these ideas we consider the matching of the five dimensional gauge coupling in the UV. In [20, 21] this coupling is matched to the perturbative result for the vector vector correlator in QCD. The AdS gravitational dual presumably describes a strongly coupled conformal theory in the UV and so the correlator behaviour matches the logarithmic result of the conformal but weakly coupled UV behaviour of QCD. It is surprising that the numerical coefficient of the log term can be matched though. Here we test how good that matching is by allowing the parameter to float and fitting it to data. We find such a fit induces roughly a 30% change in the coupling value, which provides a measure of non-perturbative corrections at the scale of matching to the strongly coupled regime of QCD. We leave attempts to further improve the UV of the theory for later work though.

Finally, it has recently been pointed out [31] that an appropriate change to the IR behaviour of the dilaton can correct the  $n$  scaling of the tower of excited  $\rho$  meson states. We have tested our model in this respect but find only a marginal improvement over the pure AdS case. This is a sign that, although our geometry describes a non-supersymmetric gauge configuration, it is still not a perfect description of QCD and work remains to be done on improving the geometric background.

## 2. Phenomenological five dimensional models

The phenomenological approaches to describing QCD holographically are based on a 5d action of the form

$$S \sim \int d^4x dr e^\phi \sqrt{-g} \left( \mathcal{L}_\sigma + \sigma^2 \text{Tr} |DU|^2 - \frac{1}{4g_5^2} \text{Tr} (F_L^2 + F_R^2) \right) \quad (2.1)$$

where  $D_\mu U = \partial_\mu U - iA_{L\mu}U + iUA_{R\mu}$ . The field  $U(x, r) = \exp(i\pi^a(x, r)T^a)$  describes the pions produced by the breaking of a  $U(N_f)$  chiral symmetry with generators  $T^a$ . We assume that the background value of  $U$  is the identity so we are studying  $N_f$  degenerate quarks. The non-abelian gauge fields  $A_L$  and  $A_R$  couple by left and right action on  $U$ . They will holographically describe the vector and axial vector mesons. The field  $\sigma$  is a function of  $r$  only and holographically describes the quark mass and  $\langle \bar{q}q \rangle$  expectation value. A non-zero value for this field will break the  $U(N_f)_L \times U(N_f)_R$  chiral symmetry of the action down to the vector  $U(N_f)_V$ .

## 2.1 Pure AdS

In the simplest approaches [20, 21], the dilaton,  $\phi$ , is taken to be constant, so drops from the action. The background metric is AdS down to some boundary at  $r_0$  which breaks the conformal symmetry and provides the theory with a mass gap.

$$ds^2 = \frac{r^2}{R^2} dx^2 + \frac{R^2}{r^2} dr^2, \quad r_0 \leq r < \infty. \quad (2.2)$$

Note that dilatation transformations in the field theory, which define the mass dimension of operators (for example if we scale  $x \rightarrow e^\alpha x$  then a scalar field of dimension one scales as  $\phi \rightarrow e^{-\alpha} \phi$ ), are mapped to a symmetry of the metric with the radial direction scaling as an energy scale.

The Lagrangian for  $\sigma$  in these models is given by

$$\mathcal{L}_\sigma = (\partial_r \sigma)^2 - 3\sigma^2, \quad (2.3)$$

with resulting solutions  $\sigma(r) = m/r + c/r^3$ . Here  $\sigma$  does not transform under the field theory dilatations so  $m$  has dimension one and  $c$  dimension three. The two parameters  $m, c$  are fitted phenomenologically to the (degenerate) light quarks' mass and condensate.

The remaining parameter is  $g_5$ , which in string theory duals is a prediction in terms of the gauge theory 'tHooft coupling  $g_{\text{YM}}^2 N$ . In the phenomenological approach though, this relation is abandoned and the value of  $g_5$  is fitted to the vector current correlator extracted from QCD.

$$\int d^4x e^{iqx} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \delta^{ab} (q_\mu q_\nu - qg_{\mu\nu}) \Pi_V(-q^2), \quad (2.4)$$

where  $J_\mu^a(x) = \bar{q} \gamma_\mu T^a q$ . For QCD, the leading order contribution to  $\Pi_V(-q^2)$  is [34]

$$\Pi_V(-q^2) = -\frac{N}{24\pi^2} \ln(-q^2). \quad (2.5)$$

In order to calculate this quantity from the five dimensional model, we appeal to the AdS/CFT correspondence. The five dimensional vector field  $V_\mu^a(x, r) = (A_{L\mu}^a(x, r) + A_{R\mu}^a(x, r))$  acts as a source for the four dimensional vector current  $J_\mu^a(x)$  in the limit  $r \rightarrow \infty$ . It obeys the equation of motion

$$\partial_\mu \left( \frac{1}{g_5^2} e^\phi \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} (\partial_\alpha V_\beta^a - \partial_\beta V_\alpha^a) \right) = 0. \quad (2.6)$$

We look for solutions of the form  $V^\mu(x, r) = V_0^\mu(x) v(x, r)$ , with  $\lim_{r \rightarrow \infty} v(x, r) = 1$ , so that  $V_0^\mu(x)$  will act as a dimension one source for  $J_\mu^a(x)$ . Solving the equation of motion (2.6) in the  $V^r(x, r) = 0$  gauge gives

$$v(q, r) = -\frac{\pi}{2} \mathcal{Y}_1(q/r) \sim 1 - \frac{q^2}{4r^2} \ln \left( \frac{-q^2}{r^2} \right), \text{ as } r \rightarrow \infty, \quad (2.7)$$

where  $\mathcal{Y}_1$  is a Bessel function of the second kind. Substituting the solution back into the action and differentiating twice with respect to the source  $V_0^\mu$  gives the vector current correlator

$$\Pi_V(-q^2) = \left[ \frac{1}{g_5^2 q^2} r^3 \partial_r v(q, r) \right]_{r=\infty}, \quad (2.8)$$

which (up to contact terms) yields

$$\Pi_V(-q^2) = -\frac{1}{2g_5^2} \ln(-q^2). \tag{2.9}$$

Finally, comparing this to the perturbative QCD result (2.5) determines the 5d coupling as

$$g_5^2 = \frac{12\pi^2}{N}. \tag{2.10}$$

In [21, 20] this model is used to calculate meson masses, decay constants and couplings coefficients with great success. We summarize these results in table 1.

The matching in (2.10) is of course naive. One should match the gravitational theory to QCD only at the point where the QCD coupling becomes non-perturbative where gluonic corrections to the perturbative QCD result become important. It is therefore interesting to recompute the results of [21], but with  $g_5$  being a free parameter of the model in order to see how accurate this matching is. On performing a global fit on all of the parameters, we found that the optimal value for  $g_5$  is 5.19 which is 17% smaller than the result  $\sqrt{(12\pi^2)/N}$  from matching to perturbative QCD. We conclude that non-perturbative effects could have a significant effect.

## 2.2 The new model

Our approach in this paper will be based around the D3/D7 brane string theory construction described in the appendix [12]. Here we will present the model as a 5d model in the spirit of (2.1). Starting with the string theory model's action (A.10), we construct a phenomenological model by artificially extending the symmetry group from  $U(N_f)_V \times U(1)_A$  to the chiral  $U(N_f)_L \times U(N_f)_R$  and add in the axial vector gauge field in (2.1).

The model has the metric

$$ds^2 = H^{-1/2} f^{-\delta/4} \sum_{i=0}^3 dx_i^2 + H^{1/2} f^{1/2-\delta/4} h dr^2, \tag{2.11}$$

where

$$f = \frac{(\sigma(r)^2 + r^2)^2 + b^4}{(\sigma(r)^2 + r^2)^2 - b^4}, \quad h = \frac{(\sigma(r)^2 + r^2)^2 - b^4}{(\sigma(r)^2 + r^2)^2}, \quad H = f^\delta - 1,$$

and a radially changing dilaton and 5d gauge coupling

$$e^\phi = H^{5/4} f^{5/4-5\delta/8+\Delta/2} h^{5/2} r^3 (1 + \dot{\sigma}^2)^{-1/2} \sim r^{-2}, \text{ as } r \rightarrow \infty, \\ g_5^2 = \hat{g}_5^2 H^{1/2} f^{1/2-\delta/4+\Delta/2} h (1 + \dot{\sigma}^2)^{-1} \sim \hat{g}_5^2 r^{-2}, \text{ as } r \rightarrow \infty. \tag{2.12}$$

with  $\delta = 1/2$ ,  $\Delta = \sqrt{39}/2$ . Note that we scaled all coordinates by a factor of  $R$ . The conformal symmetry breaking scale is fixed by the parameter  $b$  which will determine the scale  $\Lambda_{\text{QCD}}$ . Since it is the only scale in the model we set it to one for computations. At the string theory level the value of  $R$  fixes the 5d gauge coupling, but here we will fix that phenomenologically to describe an  $N_c = 3$  theory so we have also set  $R = 1$  and left  $\hat{g}_5$  free. As  $r \rightarrow \infty$ , the metric returns to  $AdS_5$ , the factor  $e^\phi/g_5^2$  goes to  $1/\hat{g}_5^2$  and we are left

with exactly the pure AdS model. The radial dependence of the dilaton shows the model has a running coupling<sup>1</sup>

**Dynamical quark condensate.** The chiral symmetry breaking quark condensate is determined dynamically in this model by the background metric which represents the background gauge configuration. The Lagrangian for the field  $\sigma(r)$  in this model is

$$\mathcal{L}_\sigma = \sqrt{-g} f^{\Delta/2} g_{rr}^{3/2} \sqrt{1 + \dot{\sigma}^2}, \quad (2.13)$$

where the dot indicates differentiation with respect to  $r$ . The equation of motion for this field, which is complicated since  $\sigma$  occurs throughout the geometry, is given by

$$\frac{d}{dr} \left[ \frac{f^{\Delta/2} \mathcal{G}(r, \sigma)}{\sqrt{1 + (\partial_r \sigma)^2}} (\partial_r \sigma) \right] - \sqrt{1 + \partial_r \sigma^2} \frac{d}{d\sigma} \left[ f^{\Delta/2} \mathcal{G}(r, \sigma) \right] = 0, \quad (2.14)$$

where

$$\mathcal{G}(r, \sigma) = r^3 \frac{((r^2 + \sigma^2)^2 + 1)((r^2 + \sigma^2)^2 - 1)}{(r^2 + \sigma^2)^4}. \quad (2.15)$$

The large  $r$  form of the solutions is of the AdS form (note from the metric that  $\sigma$  here enters symmetrically with  $r$  and therefore is rescaled relative to (3) and has energy dimension one)

$$\sigma(r) = m + c/r^2 + \dots \quad (2.16)$$

where  $m$  and  $c$  are interpreted as the the quark mass and condensate respectively. We seek regular solutions that satisfy  $\dot{\sigma}(0) = 0$ . There is a single such solution for each value of  $\sigma(0)$  indicating that the condensate  $c$  is determined for a fixed asymptotic value of  $m$ . The solutions are shown in figure 1.

Note that when the dynamical function  $\sigma(r)$  is included in the metric for the model there is no singularity since one cannot reach  $r + \sigma = b$ . The model therefore extends smoothly down to  $r = 0$ . We do not need to impose a hard IR cut off and the conformal symmetry breaking is expressed through the parameter  $b$  only.

**Matching the 5d coupling.** The matching occurs at the boundary  $r \rightarrow \infty$ , so the results are exactly the same as those for the pure AdS calculation, and we are lead to the identification  $\hat{g}_5^2 = (12\pi^2)/N$ .

**Vector mesons.** We look for solutions to the vector equation of motion (2.6) that are of the form  $V_\mu^a(x, r) = V_\mu^a(r) \exp(iqx)$ . In the  $V_r^a(x, r) = 0$  gauge this gives the following equation of motion

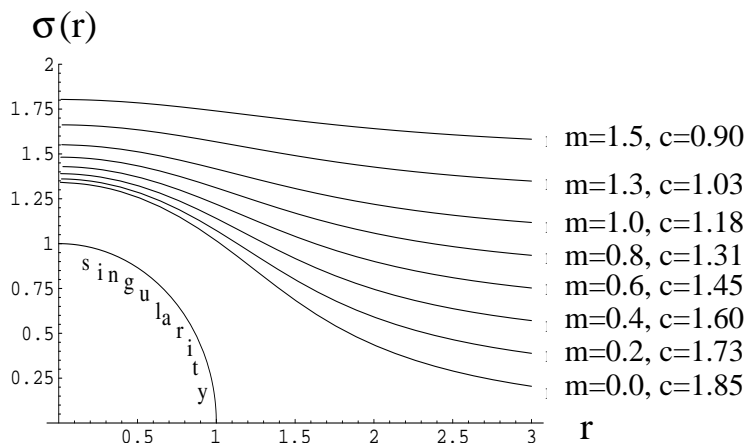
$$\partial_r(K_1(r) \partial_r V_\mu^a(r)) + q^2 K_2(r) V_\mu^a(r) = 0, \quad (2.17)$$

with

$$K_1 = f^{1/2} h r^3 (1 + \dot{\sigma}^2)^{-1/2}, K_2 = H f^{1-\delta/2} h^2 r^3 (1 + \dot{\sigma}^2)^{-1/2}.$$

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<sup>1</sup>Formally in the string theory model of the appendix the running coupling may be determined by placing a D3 brane probe in the geometry as in [36] which gives  $g_{YM}^2 \sim (w^4 + b^4)/(w^4 - b^4)^\Delta$ . Note this running in the strong coupling regime is not logarithmic and the gauge coupling diverges at the scale  $b$ .



**Figure 1:** A plot of the embedding of the D7 brane as a function of the radial coordinate  $r$

We will interpret the rho mesons as normalisable modes of this equation, with the eigenvalues corresponding to the squared rho masses  $m_\rho^2 = -q^2$ . For these modes to be normalisable, we require that they vanish sufficiently rapidly as  $r \rightarrow \infty$ . We must also impose the gauge invariant boundary condition  $\psi'_\rho(0) = 0$  to ensure the smoothness of the solution.

The rho wavefunction  $\psi_\rho(r)$  is then a solution to (2.17) for an arbitrary component of  $V_\mu^a(r)$  subject to the boundary conditions  $\lim_{r \rightarrow \infty} \psi_\rho(r) = 0$  and  $\psi'_\rho(0) = 0$ . We solve the equation numerically to find the spectrum of rho masses.

For large  $N$ , one can write the vector current correlator as the sum over rho resonances

$$\Pi_V(-q^2) = - \sum_\rho \frac{F_\rho^2}{(q^2 - m_\rho^2)m_\rho^2}, \tag{2.18}$$

where  $F_\rho$  is the rho decay constant defined by  $\langle 0 | J_\mu^a | \rho^b \rangle = F_\rho \delta^{ab} \epsilon_\mu$ . In order to find  $F_\rho$ , we proceed by finding the Green's function solution to (2.17). Imposing the completeness relation

$$\sum_\rho K_2(r) \psi_\rho(r) \psi_\rho(r') = \delta(r - r') \tag{2.19}$$

on the set of eigenfunctions one finds

$$G(q; r, r') = \sum_\rho \frac{\psi_\rho(r) \psi_\rho(r')}{q^2 - m_\rho^2}. \tag{2.20}$$

Generalising (2.8) we have

$$\Pi_V(-q^2) = \left[ \frac{1}{\hat{g}_5^2 q^2} K_1(r) \partial_r v(q, r) \right]_{r=\infty}. \tag{2.21}$$

It can be shown that, in terms of the Green's function,  $v(q, r') = [K_1(r) \partial_r G(q; r, r')]_{r=\infty}$ . From this, one finds

$$\Pi_V(-q^2) = - \frac{1}{\hat{g}_5^2} \lim_{r \rightarrow \infty} \sum_\rho \frac{(K_1(r) \psi'_\rho(r))^2}{(q^2 - m_\rho^2)m_\rho^2}. \tag{2.22}$$



Comparing this to (2.18) we can extract the rho decay constant

$$F_\rho^2 = \frac{1}{\hat{g}_5^2} \lim_{r \rightarrow \infty} (K_1(r) \psi'_\rho(r))^2. \quad (2.23)$$

**The axial vector mesons.** We write the axial vector field,  $A_\mu^a = (A_{L\mu}^a - A_{R\mu}^a)$ , in the  $A_{ar}(x, r) = 0$  gauge, as perpendicular components plus a longitudinal component  $A_a^\mu = A_{a\perp}^\mu + \partial^\mu \phi$ . The equation of motion for the perpendicular components  $A_{\mu\perp}^a$  with  $A_{i\perp}(x, r) = A(q, r) \exp(iqx)$  is

$$\partial_r(K_1(r) \partial_r A_\mu^a(r)) + q^2 K_2(r) A_\mu^a(r) - \hat{g}_5^2 \sigma(r)^2 K_3(r) A_\mu^a(r) = 0, \quad (2.24)$$

where  $K_1(r)$  and  $K_2(r)$  are the same as in (2.17), and  $K_3(r) = H f^{3/2 - \delta/2 + \Delta/2} h^3 r^3 (1 + \dot{\sigma}^2)^{-1/2}$ . The solutions represent the  $a_1$  spin 1 axial vector meson if we let  $\lim_{r \rightarrow \infty} \psi_{a_1}(r) = 0$ ,  $\partial_r \psi_{a_1}(0) = 0$ . We find the masses  $m_{a_1}^2 = -q^2$  by numerically finding the eigenvalues of this equation. The decay constant  $F_{a_1}$  is found in the same way as (2.23).

The pion decay constant is similarly given by

$$f_\pi^2 = \frac{1}{\hat{g}_5^2} [K_1(r) \partial_r \psi_{a_1}(0, r)]_{r=\infty}. \quad (2.25)$$

We can then extract the quark mass using the Gell-Mann-Oakes-Renner relation which must be obeyed for small quark masses (ignoring the  $m_q$  dependence of the condensate)

$$m_\pi^2 f_\pi^2 = 2m_q c \quad (2.26)$$

Results from these methods are displayed in table 1 and discussed in section 3.

**The pion.** The pion and longitudinal axial gauge fields mix and one must look for a solution of the coupled field equations

$$\partial_r(K_1(r) \partial_r \phi) + \hat{g}_5^2 \sigma(r)^2 K_3(r) (\pi^a - \phi^a) = 0 \quad (2.27)$$

$$-q^2 K_1(r) \partial_r \phi + \hat{g}_5^2 K_4(r) \sigma(r)^2 \partial_r \pi = 0 \quad (2.28)$$

where  $K_4(r) = f^{1 + \Delta/2} h^2 r^3 (1 + \dot{\sigma}^2)^{-1/2}$ .

The regular solutions of these equations require one to fix two unknowns, the mass of the pion,  $-q^2$ , and the ratio of the  $\phi$  and  $\pi$  fields at  $r = 0$ . This is numerically hard. Instead one can use the values of  $m_\pi$  taken from the Gell-Mann-Oakes-Renner relation and then find the ratio  $\phi(0)/\pi(0)$  with  $\phi'(0) = 0$  which leads to  $\phi(r = \infty) = \pi(r = \infty) = 0$ .

**The coupling  $g_{\rho\pi\pi}$ .** To the order we are working the value of the  $g_{\rho\pi\pi}$  coupling can be read off from the expansion of  $|DU|^2$  in the action. This is not entirely satisfactory since  $Tr F^3$  terms, which we don't include in the action, will also contribute. Nevertheless for comparison to [20] we will compute them for our best fit models below. In particular

$$g_{\rho\pi\pi} = \int dr \hat{g}_5 \psi_\rho(r) \left( \frac{K_1(r) (\partial_r \phi)^2}{\hat{g}_5^2} + \sigma(r)^2 K_3(r) (\pi^a - \phi^a) \right) \quad (2.29)$$

The  $\pi$  field is normalized so the expression in brackets in this last equation integrates to one.

Observable	Measured (MeV)	Model A (MeV)	AdS A (MeV)	Model B (MeV)	AdS B (MeV)
$m_\pi$	$139.6 \pm 0.0004$	139.6*	139.6*	139.0	141
$m_\rho$	$775.8 \pm 0.5$	775.8*	775.8*	742.7	832
$m_{a_1}$	$1230 \pm 40$	1396	1363	1337	1220
$f_\pi$	$92.4 \pm 0.35$	87.6	92.4*	83.9	84.0
$F_\rho^{1/2}$	$345 \pm 8$	310.2	329	297.0	353
$F_{a_1}^{1/2}$	$433 \pm 13$	513.1	486	491.4	440

**Table 1:** Results for meson variables in the models discussed in the text. Model A is the new model in the paper with parameters fixed to the starred measurements. AdS A is the equivalent pure AdS model results with a hard IR cut off and the value of the condensate being fitted. Model B is a global fit in the new model and AdS B is the equivalent fit result in pure AdS.

### 3. Results

The results of the model are displayed in table 1. We compute 6 QCD meson parameters for our fits (we do not include  $g_{\rho\pi\pi}$ ). Our model has two free parameters (after fixing  $g_5$  phenomenologically as discussed above),  $b$  corresponding roughly to the strong coupling scale  $\Lambda$  and  $m$  corresponding to the light quark mass. The model therefore has the same number of free parameters as real QCD.

In the first model, A, we match  $b$  and  $m$  by demanding that we correctly reproduce  $m_\pi$  and  $m_\rho$ . In order to do this, we must set  $\Lambda_b = 264.5 \text{ MeV}$  and  $m = 2.16 \text{ MeV}$ . This gives a prediction of  $325.8 \text{ MeV}$  for the scale of the quark condensate. The overall rms error for this model is 12.8% (Note  $\epsilon_{rms} = \sum_O ((\delta O/O)^2/n)^{1/2}$  with  $O$  the observable and here  $n = 4$ ). For comparison we also reproduce the pure AdS fit to the same parameters found in [20]. That model has three free parameters, the value of the IR cut off, the quark mass and the quark condensate and is therefore less predictive.

In model B, we perform a global fit to all observables. This gives  $\Lambda_b = 253.2(\text{MeV})$  and  $m = 2.24 \text{ MeV}$ , with the characteristic scale for the quark condensate  $311.9 \text{ MeV}$ . The overall rms error for this model is 11.6%. Again we reproduce the equivalent pure AdS model fit for comparison.

For the best fit point we have also computed  $g_{\rho\pi\pi} = 4.81 \text{ MeV}$  using (2.29). This should be compared to the experimental result of  $6.03 \pm 0.07 \text{ MeV}$  and to the results in [20] of  $4.48 \text{ MeV}$  although, as discussed above, the computation of this coupling is less robust than the other results.

It is again interesting to test how well determined the 5d gauge coupling  $g_5$  is by the phenomenological fit to the far UV expectation for  $\Pi_V$ . For example if one fits  $\Lambda$ ,  $m_q$  and  $g_5$  to correctly reproduce the three meson masses one finds  $g_5 = 4.36$  which is 30% lower than the value  $\sqrt{12\pi^2/N_c}$  from perturbative QCD.

## 4. Conclusions

We have adapted a string theoretic model of chiral symmetry breaking to a phenomenological description of QCD. The model we have proposed goes some way towards addressing the inconsistencies of simple AdS slice holographic QCD models [20, 21]. The background geometry of our model is non-supersymmetric, and it is the smooth variation of this geometry with the radial direction  $r$  that provides a mass gap, without the need for an artificial hard IR cut-off. In addition, the dual field to the quark mass/condensate operator is a natural part of the geometrical set-up with the value of the condensate being determined by the quark mass.

However, this is still a phenomenological approach in that we introduce extra fields and symmetries by hand into the model in order to describe the full pion and axial vector sectors. Formally there is no geometric string interpretation for this system. We also treat the background as though it describes an  $N = 3$  rather than an  $N = \infty$  field theory by matching the 5d gauge coupling to QCD.

We find that the predictions of this model match experimental results to within 12%. This model is a little more predictive than the pure AdS slice models since the condensate is dynamically determined by the geometry. The best fit is in fact a few percent worse than the AdS slice models but hopefully the theoretical improvements represent at least a moral victory. In any case one would naively have expected errors of order a few 100% in all of these models so the closeness to QCD across a range of holographic models supports the robustness of the approach.

A drawback of these models to date has been that the geometry returns to AdS for large  $r$ , meaning that the field theory is not asymptotically free in the UV. Incorrect physics in the UV will affect the strong coupling regime in the IR [33]. Here we investigated corrections to the matching of the 5d gauge coupling to naive perturbative QCD results. We found that this coupling's value should be changed at the 30% level indicating the size of non-perturbative effects. In the future one might hope to study the importance of higher dimension operators in the IR physics as well.

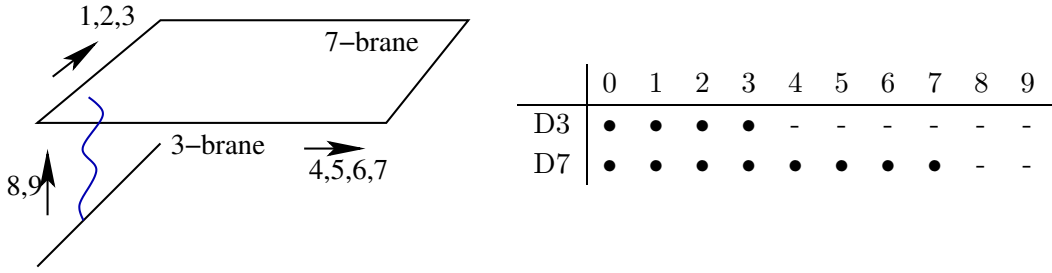
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## A. String theory progenitor

The phenomenological model used here is based on the AdS/CFT Correspondence realization of chiral symmetry breaking in [13]. That model consists of a dilaton flow deformed AdS geometry

$$ds^2 = H^{-1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^6 dw_i^2, \quad (\text{A.1})$$



**Figure 2:** The basic geometry of the D3 D7 system under consideration.

where

$$H = \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^\delta - 1 \quad (\text{A.2})$$

and the dilaton and four-form are given by

$$e^{2\phi} = e^{2\phi_0} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^\Delta, \quad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz. \quad (\text{A.3})$$

There are formally two free parameters,  $R$  and  $b$ , since

$$\delta = \frac{R^4}{2b^4}, \quad \Delta^2 = 10 - \delta^2 \quad (\text{A.4})$$

We can see that dimensionally  $b$  has energy dimension one and enters to the fourth power. The  $\text{SO}(6)$  symmetry of the geometry is retained at all  $r$ . We conclude that in the field theory a dimension four operator with no  $\text{SO}(6)$  charge has been switched on.  $b^4$  therefore corresponds to a vev for the operator  $\text{Tr}F^2$ .

Quarks are introduced by including probe D7 branes into the geometry. As shown in figure 1, strings which stretch between the D3 and D7 branes are in the fundamental representation of the  $\text{SU}(N)$  gauge theory on the D3. The length of the minimum length string between the two branes determines the mass of the quark field. We minimize the D7's world-volume in the spacetime around the D3 branes. This is encoded by the Dirac Born Infeld action in Einstein frame of the D7 brane

$$S_{D7} = -\tau_7 \int d^8\xi e^\phi [-\det(\mathbf{P}[g_{ab}])]^{\frac{1}{2}}, \quad (\text{A.5})$$

where the pull back of the metric  $\mathbf{P}[g_{ab}]$  is given by

$$\mathbf{P}[g_{ab}] = g_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b} \quad (\text{A.6})$$

Substituting from the geometry above we can find the equation of motion for the radial separation,  $\sigma$ , of the two branes in the 8,9 directions as a function of the radial coordinate  $r$  in the 4 – 7 directions. It is just eq. (2.14) with the solutions shown in figure 1. The

solutions show that a dynamical mass is formed for the quarks. A massless quark would correspond to a D7 brane that intersects the D3 brane so there was a zero length string between them. We see that the D3s repel the D7 and for all configurations there is a minimum length string. The solution which asymptotically has  $m = 0$  also explicitly break the U(1) symmetry in the 8, 9 plane by bending off the axis. This is the geometric representation of the breaking of the U(1) axial symmetry of the quarks.

Fluctuations of the brane about the solution found above in the 8, 9 directions correspond to excitations of the operator  $\bar{q}q$  and contain information about the pion and sigma field of the model. Letting  $u_8 + iu_9 = \sigma(r) U(r, \xi)$  and expanding to second order in  $U(r, \xi)$  gives

$$S = -\tau_7 \int d^8\xi e^\phi \sqrt{-g} (1 + \dot{\sigma}^2)^{\frac{1}{2}} \left[ 1 + \frac{1}{2} g_{rr} \sigma^2 (1 + \dot{\sigma}^2)^{-1} \partial^a U \partial_a U^\dagger \right]. \quad (\text{A.7})$$

Letting  $U(r, \xi) = \exp(i\pi(r, \xi))$ , this gives an action for the pion field and the  $\sigma$  field.

There is also a superpartner U(1) gauge field in the action which describes the operator  $\bar{q}\gamma^\mu q$  and hence vector mesons. This is introduced as a gauge field  $F_{ab}$  living on the D7

$$S = -\tau_7 \int d^8\xi e^\phi \left[ -\det(\mathbf{P}[g_{ab}] + 2\pi\alpha' e^{-\phi/2} F_{ab}) \right]^{\frac{1}{2}}, \quad (\text{A.8})$$

which, expanded to second order gives

$$S = -\tau_7 \int d^8\xi e^\phi \sqrt{-g} (1 + \dot{\sigma}^2)^{\frac{1}{2}} \left[ 1 + \frac{1}{2} g_{rr} \frac{\sigma^2}{(1 + \dot{\sigma}^2)} \partial^a U \partial_a U^\dagger - \frac{1}{4} \frac{(2\pi\alpha')^2}{(1 + \dot{\sigma}^2)} e^{-\phi} F^2 \right]. \quad (\text{A.9})$$

Now, if we assume that the fields do not have any components on the three sphere (which is appropriate for duals to non-supersymmetric fields) we arrive at the 5d action

$$S = -R^{-8} \int d^4x dr e^\phi \sqrt{-g} \left( \mathcal{L}_\sigma + \sigma^2 |\partial U|^2 - \frac{1}{4g_5^2} F^2 \right) \quad (\text{A.10})$$

Between (A9) and (A10) we have rescaled  $U \rightarrow R^4 \pi \sqrt{\tau_7} U$  and redefined the metric, dilaton and  $g_5$  to the appropriate notation for a 5d model. In particular  $g_5$  and the dilaton now have additional  $r$  dependence which is just that found in (12). The asymptotic large  $r$  value of  $g_5$  is given by  $16\pi^3 g_s \alpha'^2 / r^2$ . Normalizable solutions of the fields in this model in the fifth direction correspond to physical states in the gauge theory, with the quantum numbers of the operators described by the holographic field. Integrating over  $r$  then leaves the four dimensional effective Lagrangian for these states from which masses and couplings can be read off. A more complete analysis of this model can be found in [12, 13, 35]. This method can be extended to give a theory with an  $U(N_f)_V \times U(1)_A$  symmetry by replacing the single D7 brane with a stack of  $N_f$  D7 branes (at large  $r$  the theory becomes supersymmetric and there is a superpotential term linking the adjoint matter fields and the quarks which breaks the  $U(N_f)_A$  symmetry). We must, however, be careful to keep  $N_f \ll N$  so that we can still treat the stack of D7 branes as a probe, and ignore any back reaction on the geometry.

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